

# International Journal of Innovative Research in Science, Engineering and Technology

(A High Impact Factor & UGC Approved Journal)

Website: [www.ijirset.com](http://www.ijirset.com)

Vol. 6, Issue 8, August 2017

## A Review on Graphical Models of Complex Network

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**ABSTRACT:**Complex networks describe a wide range of systems in nature and society. The complex network study is a relatively recent field, and it has been inspired by the observation of many real systems, such as biological, social or technological ones. The complex networks are a set of many connected nodes that interact in different ways. The nodes in a network are also called vertices or elements  $V$ , where  $N$  is the total number of nodes in the network. Recently, it has been empirically observed that, besides the spatially or topologically structured organization, the temporal constraints imposed on many complex interacting systems add a further dimension crucial for understanding their generic structure and dynamics. This paper reviews the structure and graphical models of complex networks.

**KEYWORDS:**Complex network, Random graph, Scale free graph, Graph theory, Power-Law degree distribution

### I. INTRODUCTION

Many real world systems can be described as complex networks. Complex networks exist in many different fields of disciplines in the natural and social sciences as well as in nature. Models of complex networks can be used to describe many systems in nature. A complex network represents the interactions of subjects and objects in a complex system. A Complex network is a network with non-trivial topological features, with patterns of connection between their elements that are neither purely regular nor purely random. Its structure is irregular, complex and dynamically evolving in time. Typical examples of complex systems are social systems and any type of biological cells. Frequently cited examples of complex networks include the Internet, the World Wide Web, power grids, transportation systems, food webs, ecosystems, genetic networks, neural networks, social networks, epidemiology networks, etc. On the Internet, nodes are routers (or domains) connected by physical links such as optical fibers. The nodes of the WWW are webpages connected by directed hyperlinks. These networks contain non-trivial network structure. Any natural or social structure, including those undergoing dynamic changes of the network topology can be described by complex network. To clearly understand and predict the overall network performance, a proper knowledge of networks' structure is required. The study on complex networks define new concepts and measures to characterize the topology of real networks.

Paper is organized as follows. Section II describes the structure of complex network. Models of complex network is described in Section III. Section IV explains the applications of complex network. Finally, Section V presents conclusion.

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## II. THE STRUCTURE OF COMPLEX NETWORKS

### • Definitions and notations

Complex networks and graph theory are closely intertwined areas. The study of complex systems is rooted in the theories and methods originally developed in the field of mathematical graph-theory and statistical physics. Graph theory has a very wide range of applications in engineering, in physical, social, and biological sciences, in linguistics and in numerous other areas. A graph can be used to represent almost any physical situation involving discrete objects and a relationship among them[6]. A graph is the mathematical abstraction of a network. Formally, a graph is used to represent a complex network. An undirected (directed) graph  $G = (V, E)$  consists of two sets  $V$  and  $E$ , such that  $V \neq \emptyset$  and  $E$  is a set of unordered (ordered) pairs of elements of  $V$ . The elements of  $V \equiv \{v_1, v_2, \dots, v_N\}$  are the nodes (or vertices) of the graph  $G$ , while the elements of  $E \equiv \{e_1, e_2, \dots, e_K\}$  are its links (or edges, or lines).  $N$  is the total number of nodes of the network, and  $K$  is the total number of links. An individual node of the network is denoted by a label  $i$  in the set  $V$ . In an undirected graph, each of the links is defined by a couple of nodes  $i$  and  $j$ , and is denoted as  $(i, j)$ . In a directed graph, the order of the two nodes is important:  $lij$  stands for a link from  $i$  to  $j$ , and  $lij \neq jli$ . [5]

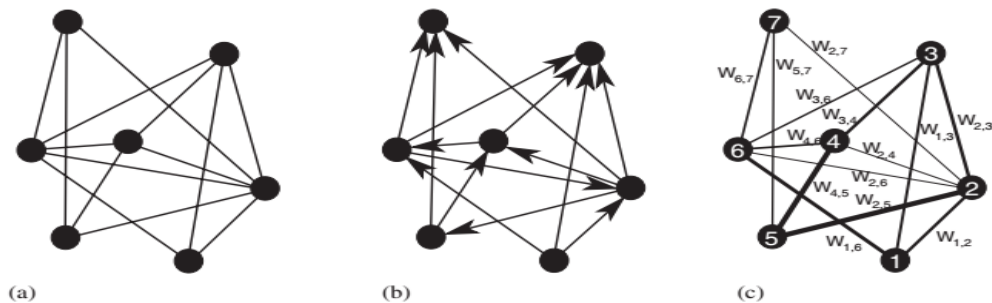


Fig.2.1 Graphical representation of an undirected (a), a directed (b), and a weighted undirected (c) graph with  $N = 7$  nodes and  $K = 14$  links. In the directed graph, adjacent nodes are connected by arrows, indicating the direction of each link. In the weighted graph, the values  $w_{i,j}$  reported on each link indicate the weights of the links, and are graphically represented by the link thicknesses. This figure is taken from [2].

A network is easily represented by using adjacency matrix. Given a graph  $G = (N,L)$ , the adjacency matrix  $A_{ij}$  is a  $N \times N$  square matrix, whose entry  $a_{ij}$  ( $i, j = 1, 2, \dots, N$ ) is equal to 1 when the link  $lij$  exists, and zero otherwise.

### • Node degree, degree distributions

Degree  $k_i$  of a node  $i$  is defined as the number of edges connected to a node  $i$  [1]. A directed graph has both an in-degree and an out-degree for each vertex, which are the numbers of incoming and outgoing edges respectively. For example, in a friendship network, the degree of a person is the number of friends the person has. In a railroad network, the vertices are stations and there is an edge between two stations when they are connected by a railway line.

The most fundamental topological characterization of a graph  $G$  can be obtained from its degree distribution  $P(k)$  [1] defined as the probability that a node chosen uniformly at random has degree  $k$  or, equivalently, as the fraction of nodes in the graph having degree  $k$ .

### • Shortest path lengths, diameter and centrality

In the transport and communication within a network, the concept of shortest paths play a significant role in the characterization of the internal structure of a graph. For example, send a data packet from one computer to another through the Internet: the geodesic provides a best path way. Diameter of a network is defined as the length (in number of edges) of the longest geodesic path between any two vertices. Centrality identifies the most important vertices within a graph. Degree centrality is the simplest measure of node connectivity. Degree centrality counts the number of links

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held by each node. Betweenness is useful for analyzing communication dynamics. Betweenness centrality measures the number of times a node lies on the shortest path between other nodes. This measure shows which nodes act as 'bridges' between nodes in a network. This measure scores each node based on their 'closeness' to all other nodes within the network.

- **Average path length**

Average path length is defined as the average number of steps along the shortest paths for all possible pairs of network nodes[1]. The effectiveness of information or mass transport on a network depends on average path length. One example is the average number of friends existing in the shortest path connecting two persons in the network. Average path length is calculated by finding the shortest path between all pairs of nodes, adding them up, and then dividing by the total number of pairs. This shows us, on average, the number of steps it takes to get from one member of the network to another. Diameter is the longest of all the calculated shortest paths in a network. In a real network like the World Wide Web, a short average path length facilitates the quick transfer of information and reduces costs. The efficiency of mass transfer in a metabolic network can be judged by studying its average path length. A power grid network will have less losses if its average path length is minimized.

- **Clustering, or transitivity**

Clustering of a node, is a measure of how many triangles are there on the graph, or in other words, how likely is that, if a node  $i$  has two neighbors, say  $j$  and  $k$ , then the nodes  $j$  and  $k$  are also linked to each other[1]. In graph theory, a clustering coefficient is defined as the degree to which nodes in a graph tend to cluster together. The clustering coefficient<sup>1</sup> of an undirected graph is a measure of the number of triangles in a graph.

### III.MODELS OF COMPLEX NETWORK

Traditionally, random graphs are used to model complex networks. The two other most well-known examples of recently introduced complex network models are those of small-world and scale-free graphs.

- **The random graph of Erdos and Renyi**

During the 1950's the famous mathematician Paul Erdős and Alfred Rényi put forth the concept of a random graph. It is the best studied models of a network. This model consists of  $n$  nodes or vertices, joined by links or edges which are placed between pairs of vertices chosen uniformly at random.  $G_p(N)$ , where  $N$  is the number of nodes in the graph and  $p$  is the probability of having a link between any two nodes (or shortly the link probability).  $G_p(N)$  is the set of all such graphs in which a graph having  $L$  links appears with probability  $p^L (1 - p)^{L_{max} - L}$ , where  $L_{max}$  is the maximum possible number of links. Many properties of the random graph are known analytically in the limit of large graph size  $N$ . The most important, though, is that it possesses a phase transition: from a low link density or low  $p$  value for which there are few links and many small components to a high link density or high  $p$  value for which an extensive fraction of all nodes are joined together in a single giant component.[2]

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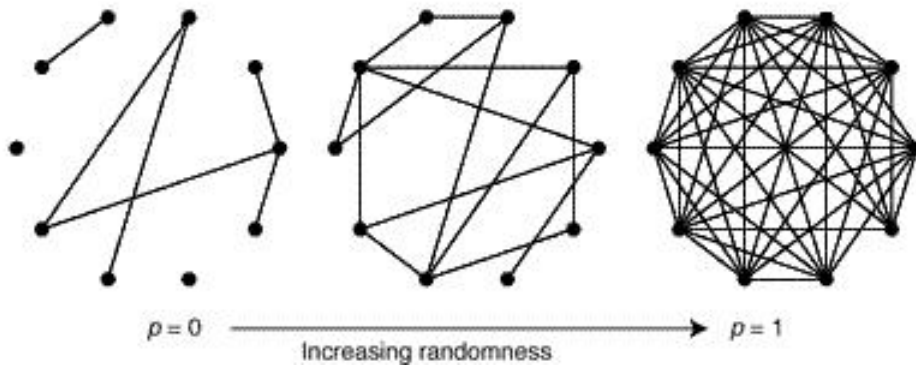


Figure 3.1 : The random graph: increasing link probability  $p$  implies that we move from a low link density for which there are few links and many small components to a high link density for which an extensive fraction of all nodes are joined together in a single giant component. This figure is taken from [5].

- **Small-World graph of Watts-Strogatz**

The "small-world effect" refers to the phenomenon that, in many real-world networks, the typical distance between nodes remains surprisingly small even as the network grows to be very large. Prior to the work of Watts and Strogatz, the "small-world effect" had been observed empirically, for example in Stanley Milgram's famous experiment on social networks [5]. Milgram's work led to the popular notion that people are typically separated by something like "six degrees of separation" in the network of human social acquaintances.

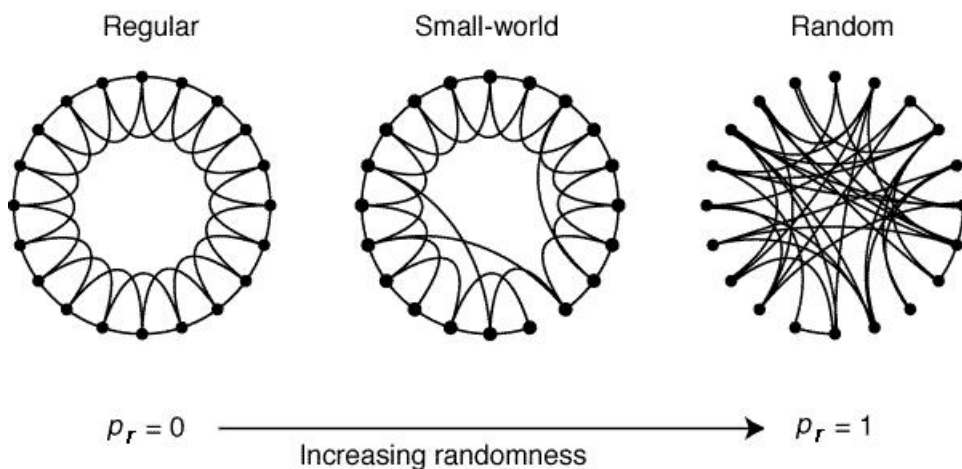


Figure 3.2 : The small-world graph: increasing the rewiring probability  $p_r$  implies that we move from the completely ordered graph ( $p_r = 0$ ) to the completely random graph ( $p_r = 1$ ). This figure is taken from [5]

It starts by constructing the ring  $R_N$  with  $N$  nodes and then linking each node to  $2s$  neighbors ( $s$  on either side of the ring). This creates a ring lattice  $R_{N,s}$  with  $L = sN$  links. The small-world graph is then created by moving, with probability  $p_r$ , one end of each link (connected to a clockwise neighbor) to a new node chosen uniformly in the ring

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lattice, except that no double links or loops are allowed. This process introduces  $p_r N$  long-range edges and allows the small-world model to interpolate between a regular lattice ( $p_r = 0$ ) and something which is similar, though not identical, to a random graph ( $p_r = 1$ ). For already small  $p_r$  the small-world becomes a locally clustered network in which two arbitrary nodes are connected by a small number of intermediate links.

- **Scale-Free graph of Barabasi-Albert**

A network that has a power-law degree distribution, regardless of any other structure, is called a scale-free network.

### Power-Law degree distribution

The degree of a node is the number of links adjacent to it. If we call the degree of a node  $k$ , a scale-free network is defined by a power-law degree distribution, which can be expressed mathematically as  $P(k) \sim k^{-\gamma}$

From the form of the distribution it is clear that when:

- $\gamma < 2$  the average degree diverges.
- $\gamma < 3$  the standard deviation of the degree diverges.

It has been found that most scale-free networks have exponents between 2 and 3. Thus, they lack a characteristic degree or scale, and therefore their name. High degree nodes are called hubs.

### The Barabasi-Albert model

The Barabasi-Albert model (a.k.a. BA model) introduced in 1998 explains the power-law degree distribution of networks by considering two main ingredients: growth and preferential attachment. The algorithm used in the BA model goes as follows.

1. Growth: Starting with a small number ( $m_0$ ) of connected nodes, at every time step, we add a new node with  $m (< m_0)$  edges that link the new node to  $m$  different nodes already present in the network[1].
2. Preferential attachment: When choosing the nodes to which the new node connects, we assume that the probability  $P$  that a new node will be connected to node  $i$  depends on the degree  $k_i$  of node  $i$ , such that

$$P \sim k_i / \sum_j k_j$$

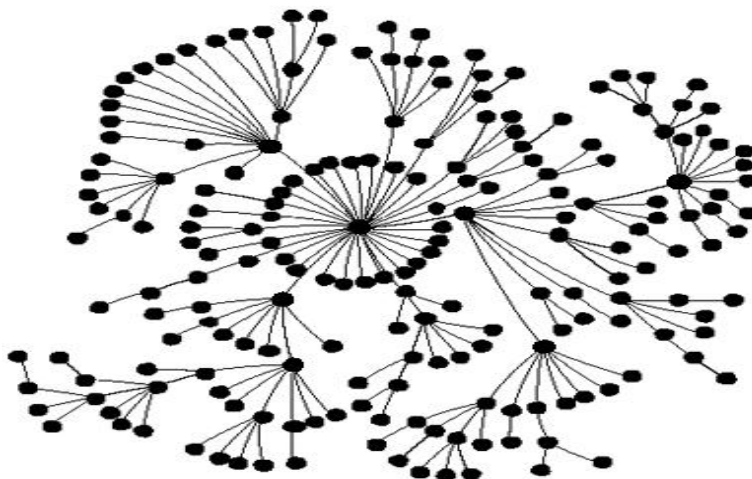


Figure 3.3 : Scale-free graph: characteristic feature of these structures is that some nodes have a degree that is orders of magnitude larger than the average value.



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## IV. APPLICATIONS

1. Biological Networks: The interaction between many biological organisms inside a cell is interesting for researchers in Biology. These biological interactions are usually modelled as Complex Networks. Metabolic Pathway networks and the Protein-Protein Interaction networks are well-studied examples.
2. Social Networks: The raise of Computational Social Science by means of high-performance computing facilities during past decade opened many windows to the computational analysis of social behavior of massive populations, however Social network Analysis itself is not that new. A lot of researches have been done in this field in order to understand the underlying process behind the behavior of large groups of people e.g. on Facebook, Tweeter or in academic collaborations.
3. Brain Networks: Brain is a complex system in which the functional connectivity of different regions can be modeled as a graph which has been shown to have small-world and scale-freeness properties. This topic nowadays is one of hot topics in Network Science.

## V. CONCLUSION

Complex networks are mostly used for modelling real world systems. A wide range of systems in nature and society are explained by complex network. This new approach provides tools for unsolved problems. The analysis of complex networks plays an important role in science. Theories and methods originally developed in the field of mathematical graph-theory and statistical physics are used for studying complex networks. In this paper, we studied a brief review on the graphical models of complex networks.

## REFERENCES

- [1] Xiao Fan Wang and Guanrong Chen, "Complex Networks: Small-World, Scale-Free and Beyond", IEEE Circuits and systems magazine 1531-636X/03/\$17.00©2003 IEEE
- [2] S. Boccaletti, V. Latora, Y. Moren, M. Chavez, D.-U. Hwang, "Complex networks: Structure and dynamics", Physics Reports 424 (2006) 175–308
- [3] D.J. Watts and S.H. Strogatz, "Collective dynamics of small-world networks", Nature, 393:440–442, 1998.
- [4] Reka Albert and Albert-Laszlo Barabasi, "Statistical mechanics of complex networks", Reviews on modern physics, Volume 74, JANUARY 2002
- [5] J. Ponce de Casasnovas, "Some Basic Concepts on Complex Networks and Games", Evolutionary Games in Complex Topologies, Springer Theses, 9, DOI: 10.1007/978-3-642-30117-9\_2, © Springer-Verlag Berlin Heidelberg 2012
- [6] Narsingh Deo, "Graph Theory with applications to engineering and computer science", Eighth Edition, Prentice-Hall Publication